MATH 12H – TOPICS IN CALCULUS – 3470 Full year - 1 credit
Grade 12
Prerequisites: Completion of Math B Regents and Honors Policy Applies

This full year course in the study of Calculus is designed to allow students the experience of college level mathematics without the detailed rigor (and stress) of the Advanced Placement program. The syllabus is very similar to that of the AB Calculus course; but topics are not covered in sufficient depth to enable students to take the AB Calculus Advanced Placement examination. The typical Math 12H student will have scored an 85 or higher on the Math B Regents, Algebra II/ Trigonometry Regents or equivalent. Contained in this honors course are topics such as: limits, differentiation, elementary integration, and their applications. The use of a graphing calculator is an integral part of this course. A school constructed examination will be administered at the midterm and at the completion of the course. Math 12H is not a replacement course for Calculus 1H.

General Department Philosophy

The Garden City Mathematics Department presents courses that align with either the New York State Regents curriculum or the College Board's Advanced Placement curriculum. In either case, the course material prepared by the Department (Grades 6 – 12) is fully consistent with these standards. In particular, our Advanced Placement syllabi have been approved by the College Board. Our Regents courses address the five NYS content strands as well as the five process strands. Our instructional activities are created to promote written and verbal mathematical communication and critical thinking skills that employ accurate mathematical ideas. The Department develops student assessments that are also consistent with the New York State and/or College Board assessment in both style and content. The scoring rubrics employed by the Department are modeled after the particular associated scoring guides. Additional information about the NYS Mathematics program can be found at http://www.emsc.nysed.gov/ciai/mst/math.html and Advanced Placement program at http://apcentral.collegeboard.com.

Members of the Department encourage their students to explore, discover and question the many fundamental concepts developed within each courses. The primary objective is to engage our students in lessons that are meaningful, inspiring and enjoyable and promote a greater interest in mathematics – at the post secondary level and beyond. Technology applications, such as calculator usage, are incorporated as developmentally appropriate and as specified by the NYS and/or College Board curriculum. The Department wants each student to realize that they can make a contribution to their class and that others can benefit from their input. The Department wants all students to maximize their mathematical potential as we move through the challenging curriculum and prepare to master all course requirements.

	Content	**Skills**
Pre-Calculus: Review of Functions	A. Domain, Range, Mapping B. Interval, Inequality, Number Line Notations	•vertical line test •translate from one type of interval notation to another •describe domain and range of relation from

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	C. Evaluating a Function D. Algebra Of Functions E. Composition of Functions	given graph •describe domain of function from given rule • combine given functions via + - × ÷ and composition
Pre-Calculus: Additional Function Theory	A. Equivalent Functions B. Odd and Even Functions C. Piecewise Functions D. Inverse Functions	•demonstrate whether or not 2 given functions are equivalent •demonstrate that a function is odd, even, or neither •use symmetry in the graph to identify odd and even functions •graph a piecewise function •write a piecewise definition from a given graph •given a function f(x), determine $f^{-1}(x)$. Verify results.
Pre-Calculus: Review of Analytic Geometry	A. Distance 1. congruent segments 2. radius of a circle 3. equation of a circle B. Midpoint C. Slope 1. parallel 2. perpendicular 3. collinear points 4. increments D. Equations of a Line 1. slope-intercept 2. point-slope E. Transformations on the X-Y plane	•write a coordinate geometry proof •use the point-slope form with ease to write the equation of a given line •reflect (x,y) in x-axis, y-axis, y=x, any horizontal or vertical line, origin •translate (x,y) •rotate(x,y) multiples of 90° •dilate(x,y)
Pre-Calculus: Absolute Value	A. Absolute Value as a Piecewise Function B. Absolute Value as a Distance C. Absolute Value as a Transformation	•rewrite an absolute value functions as a piecewise function •solve an absolute value inequality graphically using the distance concept •given the graph of f(x), draw the graph of /f(x)/
Limits: Graphic Approach	A. reality vs. expectation B. one-sided vs. two-sided limits C. types of discontinuity	•distinguish between f(a) and lim f(x) as x approaches "a" •evaluate a limit from the graph •identify point, jump, and infinite

	Content	Skills
	D. vertical and horizontal asymptotes E. limit definition of continuity	discontinuities •justify continuity using the limit definition
Limits: Evaluation Techniques	A. direct substitution B. factoring C. conjugate multiplication D. sketch near the point E. limits for a piece-wise function	•produce a "hat function" via factoring •factor the difference of perfect cubes •recognize the "shadow" pattern for conjugate multiplication •recall the sketch of a standard function to evaluate a limit •"completing" the rule for a piece-wise function to insure continuity
Limits at Infinity	A. "Behavior" at infinity 1. constant 2. cluster; converge 3. scatter; diverge B. Function of dominant terms	•recognize the 3 patterns of behavior as x approaches infinity •produce the function of dominant terms to evaluate a limit at infinity •relate the limit at infinity value to the horizontal asymptote
Curve Sketching with Limits	A. Functions with a horizontal asymptote, a vertical asymptote, and a point discontinuity B. Functions with a horizontal asymptote, a vertical asymptote, but no point discontinuity C. Functions with point discontinuities, but no asymptotes (whose "hat" function is a known graph	•incorporate a complete limit analysis to sketch the graph of a rational function
The Derivative	A. Slope of a curve B. Definition of the Derivative C. The simple Power Rule D. Manipulating exponents before and after applying the power rule	• "Fit" a tangent line to the curve to judge the slope of the curve at that point •Use the definition of the derivative on a polynomial function • Use the power rule to find the derivative of an applicable function. •express the derivative without using fractional or negative exponents
Rules for	A. Power Role	• use the correct rule to find a derivative

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Evaluating a Derivative	B. Product Rule C. Quotient Rule D. Chain Rule E. Derivative of sine and cosine functions	• recognize a function that requires more than one rule, and apply those rules appropriately
Higher Order Derivatives	A. "generations" of derivatives. B. derivative notations (f', y', dy/dx, f'', d^2y/dx^2 , etc.) C. The derivative as a "rate of change"	• repeat the derivative process on a computed derivative to obtain the "next" derivative, using the appropriate rule • introductory understanding of concavity, velocity, and acceleration
Applications	A. tangent lines and normal lines B. stationary values, inflection points C. "Mystery" functions	• write the equation of the tangent (and normal) line to a curve at a given point • use the first and second derivative to test direction and concavity • use given clues about $ax^3 + bx^2 + cx + d$ to find a, b, c, d
Rolle's Theorem	A. continuity vs. differentiability B. necessary conditions for Rolle's Theorem C. verifying Rolle's Theorem	• from a given sketch, judge continuity and differentiability at a given point •be able to state Rolle's Theorem • be able to formally justify Rolle's Theorem, or explain why it does not apply to a given function
Mean Value Theorem	A. necessary "conditions" for the Mean Value Theorem B. verifying the Mean Value Theorem	•be able to state the Mean Value Theorem •be able to formally verify the Mean Value Theorem, or explain why it does not apply to a given function
Semester Review	3 days of review, summary and synthesis.	
Curve Sketching with Derivatives	A. Stationary values, max/min, inflection points B. The First Derivative Test for max/min C. The Second Derivative Test for max/min D. Complete Derivative Analysis for curve	• be able to locate stationary values • classify a stationary value as max, min or neither • describe intervals for which a function is increasing, decreasing • locate points of inflection • test the concavity of a curve at a given point • describe intervals of concave up, down

	Content	Skills
	sketching	• sketch a curve via complete derivative analysis
Particle Motion	A. Time, as the variable. The location function B. direction, velocity, acceleration C. Path of motion. Stationary values, change in direction, total distance travelled D. average velocity, velocity vs. speed	• locate the position of an object at time t • compute average velocity on an interval • describe the velocity and acceleration at time t • determine times when the object is at rest • sketch the object's path of motion for a given interval • determine intervals when the speed is increasing or decreasing
Max/Min Problems	A. Number relation problems B. Area, perimeter, volume problems C. inverse variation/product type	• define an object function from the problem context • employ the strategy of setting the derivative of the object function = 0 • recognize extraneous solutions
Chain Rule for Parametric Equations	A. explicit vs. implicit functions B. dy/dx = dy/dt • dt/dx	• use composition to create an explicit function from 2 functions with a linked variable • find the derivative directly from the composition of parametric equations • use the chain rule on the parametric equations to get the derivative of the implied function
Implicit Differentiation	A. equations in two variables that "imply" one variable is a function of the other B. differentiating with respect to the "variable in control" C. differentiating with respect to an "outside variable" D. finding the second derivative "implicitly"	• be able to view any variable as an "implied function" of the variable in control. • appreciate that implicit differentiate is an application of the Chain Rule
Related Rates	A. TIME as an "implied variable in control" B. Pythagorean Theorem type	• recognize the related rate problem by "type" • appreciate that related rate problems are applications of implicit differentiation

	Content	Skills
	C. Proportion type D. Volume of Sphere type E. Volume of Cone type	
Antiderivatives	A. f(x), f '(x) , F(x)+ C (descendant vs. ancestor) B. the Power Rule for Antiderivatives C. Antiderivatives and particle motion D. finding the value of C	• given velocity, be able to find acceleration and location • know when, and when not, to use C
Integrals	A. Integration: the process and the notation B. The Fundamental Theorem of the Calculus C. Definite and Indefinite Integrals D. Integration by Substitution of Variable E. Integration by Parts (time permitting)	• evaluate indefinite and definite integrals, using proper technique and notation • make an algebraic substitution of variable to evaluate an integral
Areas	A. Area between a single curve and the x-axis 1. Area associated with curves falling above and below the x-axis. B. Area between 2 curves	• set up and evaluate an integral to compute area • recognize when an integral will have a negative value and compensate for the area accordingly
Volumes	A. Solids of Revolution B. Volume by Disk C. Volume by Washer D. Volume by Shell (time permitting)	• understand that rotating an area about a fixed axis produces a solid • appreciate that a volume is an infinite sum of areas • recognize when to use a disk and when to use a washer to represent a "sample" cross-section of area • be able to compute the volume when the resulting integral is of reasonable difficulty • be able to "set up", but not necessarily compute, the integral that represents the volume of a solid of revolution, including the appropriate limits of integration, as well as the function to be integrated expressed solely in terms of the variable of integration

Content	Skills

Primary Resources

Calculus: Graphing, Numerical, and Algebraic
Finney, Demana, and Kennedy
Addison Wesley

Calculus: Brief Edition
Anton, Howard
New York: John Wiley & Sons, Inc., 1995

Calculus of a Single Variable
Larson, Ron, Bruce H. Edwards, and Robert P. Hostetler.
Boston: Houghton Mifflin, 2002.

The Calculus with Analytic Geometry
Leithold, Louis
New York: Harper & Row, Publishers, 1990.

Calculus
Stewart, James
Belmont, CA: Thomson Learning, Inc. 2003.